

A Computerized Method to Generate Complex Symmetric and Geometric Tiling Patterns

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Abstract. This article contributes to a novel computerized and heuristic algorithm to make pretty geometrics used in traditional building patterns and tiling. Starting with some basic sketch or an ornamental pattern, it depicts how a designer may analyze the sketch structure and approaches to an analytical representation. In return, this analytical representation could be used to produce complicated pictures and schemes. The presented algorithm is capable of producing various complicated symmetric patterns as well as ancient Islamic geometric patterns that are called Star and Bracelet. Utilizing and expanding such patterns, the algorithm would be able to generate wide variety of divergent sketches for both monochrome and coloured patterns. The algorithm has been successfully performed and tested on the implemented software-GeoPat. Finally, the method has been applied on different types of sample patterns and the interesting results have nicely been depicted.

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1 Introduction

Innovative merging of geometric shapes and patterns is the most impressive feature of traditional buildings, especially in ancient Islamic and Greek era. All master artists in the field aver that having a good sense of analytical geometrics is an essential to gain a successful design so to reach a grand tiling pattern. Recent studies prove the important role of mathematicians who have taught analytical geometric topics to architect designers; it reveals that they virtually played a significant role to develop overall visage of both ancient and new buildings. In this regard, medieval Islam literature is the major source to study and find the relations between mathematics and architecture.

Actually, geometric patterns consist of the combination of some basic diagrams such as triangles, tetragons, pentagons, octagons and stars. Repetition, rotation and combination of such basic shapes along with specific patterns such as herbal and animalistic patterns can create beautiful forms and lattices.

These forms can numerously be found in majority of Islamic artistic works as in architectural ornamentation, consisting of tiling, plaster work, mirror work, Muqarnas¹ or stalactite work (Fig. 1.) and brick works, as well as traditional arts like metal work, lattice work, textiles, and wooden windows.

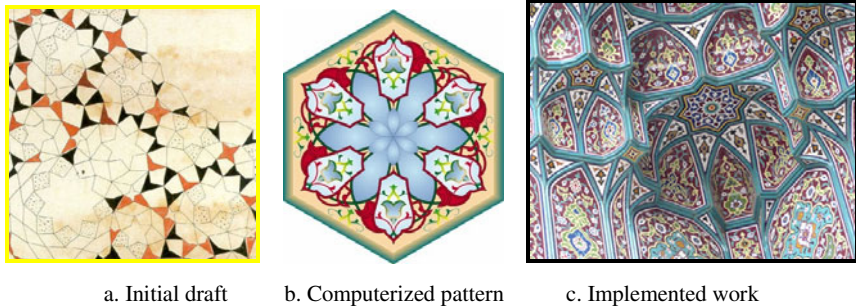


Fig. 1 Some samples of Muqarnas (stalactite work)

Islamic Geometric Patterns, especially star patterns, have stunning artistic geometric forms. These patterns are known as one of the most beautiful regular figures, have ever been made by human and that is why they are still applying in handmade buildings. As a whole, the existing Islamic patterns are taken from natural elements and phenomena. Consequently, such patterns are nominated and classified by emphasizing the elements or natural phenomena from which they are taken.

Among Islamic patterns, “*star patterns*” are one of the most famous traditional ornamental designs in the world. These patterns are a concord combination of art, mathematics, spirituality, symmetry, balance, and creativity [6, 9] which some

¹ Muqarnas is a type of corbel used as a decorative device in traditional Islamic and Persian architecture.

researchers have attended and developed ideas for the production of these patterns by computer modeling.

With the exact look to star patterns, some precise and complicated mathematical secrets can be revealed; however, as the time has passed, most of the designing techniques been gone. A complicated ancient architectural is underlying such amazing geometrics and the original designers have kept their techniques secret. On the other hand, the final implemented works just give a small amount of information about such complex patterns as well as the hidden ideas behind the layout.

Now re-finding these innovative techniques is a concern for today's researchers in the field; we hope a gradual search and process among ancient artists and mathematicians' notes and works lead us to some improvements. In this regard, some of active scientists have found some parts of these lost techniques [11, 19, 20]; however, there are still a lot to do.

In this article, we will develop a robust algorithm to gradually make several samples of the mentioned patterns which can provide several Islamic geometric patterns. In the created sketches by the algorithm, some patterns like "bracelet", different kinds of "Shamseh" and "Star" will come into view. Starting with simple implementations and then expanding to complex patterns along with the possibility to create new figures are the superior characteristics of the proposed algorithm and software.

2 Related Works

In Grunbaum and Shephard's method [13], a deep and complete program has been used on group theory to study periodic star patterns. They derived a strong set of mathematical tools to analyze patterns in symmetric groups and periodic groups, while patterns are manipulated on the surface, skillfully.

Ostromoukhov [6] has extended Grunbaum and Shephard's analysis in 17 wall-paper group styles and some stapes are contributed for artists and designers to make symmetric ornaments. In Abas and Salman [10] method, the group theory is transferred to a library covered by historic patterns. In another work [12], they are contributed the increasing distinct patterns from available mathematical tools for industrialists who made them. In all above cases, the research is extensively analytical and the proposed methods often describe how the new star patterns could be made.

Kaplan and Salesin [4] are made new sketches by filling the tiles with some pieces of star patterns on the surface of tiling. They give a surface of elements which can accept the parameters, drawn in historic samples. After that, using a deductive algorithm, they try to complete the provided sketch by filling the irregular polygons. Jay Bonner [8], an architect who has extensively studied on Islamic Star Patterns, extends a symmetric performance of star patterns on an extended surface of tiling. He is also the creator of Geodazzlers [3] which is a set of foldable papers decorated with star patterns. For a better illustration, several techniques are shown in [7]. Bonner's work is considered as a reference for designers, but it is not appropriate for software designers. Dewdney offers a complete method for "constructing designs" based on the reflection of lines through a regular order of

circles [11]; however, this technique just could be used to construct some famous but limited patterns. Dewdney realizes that his method needs several natural leaps to reach the last pattern. Similar to Dewdney's method, Arabesque software [14] allows the users to construct star patterns.

In [5], Lee shows that a pattern should be extended by drawing pieces of lines from the middle of tiling edges. He illustrates polygons-in-contact technique. On the basis of Lee and Hankin works [9], Kaplan develops some software tools for tiling [1]. M. Valiente et al. [15] performs a calculation model to analyze samples of periodic patterns. This model firstly finds the contributed objects in the primary digital picture, and then compares them with each other, and finally calculates their symmetries relations.

Lu and Steinhardt [18] found out a wide usage of subdivisions in traditional Islamic design systems e.g. overlaid Penrose kites and darts on Iranian designs. P. Rasouli et al. [19] presented two algorithms to produce computerized Islamic geometrical patterns, called 8-ZOHREH and 8-SILI. Provided though their algorithm is pretty simple, the diversity of outputted patterns is rather limited. And recently, Cromwell [20] has described a tiling-based method to construct Islamic geometric designs.

Diving all top mentioned methods, it can be inducted that the majority of them are developed based on custom tiling technique, and in order to create an especific pattern, relying on the experience of a tile worker is needed. In contrast, our recommended method does not suffer from such limitations and can perform various kinds of heuristic patterns, individually.

3 Proposed Method: Creation of Key Symmetry Patterns

As can be seen in the following pictures, Islamic patterns are some extended forms of geometric patterns, whether simple or complicated. In this method, we determine "*fundamental area*" repeated in the entire sketch so we create the entire sketch by drawing the fundamental area in a simple way, using geometric technique. Finding the fundamental area or the "*fundamental region*" is not a new problem. Lalvani [16] mentioned this problem in 1989; however, the important problem is the way of finding the fundamental area.

The initial and basic step of the suggested method produces the figures just by a independent variable parameter. First, this independent variable determines an angle of a tetragon in which the fundamental area can be settled. Then the method is expanded for tiling techniques that will be discussed later in the paper. Regarding tiling there are some interesting works has been done before by Grunbaun and Shephard in their book: "Tiling and Patterns" [21].

There are different ways to extend and transfer a pattern on a surface. In order to extend a pattern in a symmetric manner, sometimes the pattern may change or not, depending on the type of symmetry which is used. As a whole, various kinds of symmetry can be divided into four groups, including *transfer symmetry*, *reflective symmetry (mirror symmetry)*, *periodic symmetry*, and *compound symmetry*.

Different kinds of symmetries are discussed in [2], and we have used them to produce the entire sketch as well. Symmetry means balance or repetition of a part or a similar form. But, symmetry extension is much more than using it in architecture and beautiful patterns [12]. Group theory in one-dimensional symmetry indicates that periodic patterns can be analyzed in seven different ways. More information to determine the sort of practical symmetries could be found in [10].

Similarly, in two-dimensional symmetrical periodic patterns, there are seven-teen sorts of different patterns can be determined and formed [2].

3.1 Sketch Analysis within the Fundamental Region

Considering the sample sketch in fig. 2, it can be seen that there is a pattern in this sketch, the entire of which is made by the repetition of this pattern. This pattern can be put in a tetragon with the length of 'x'.

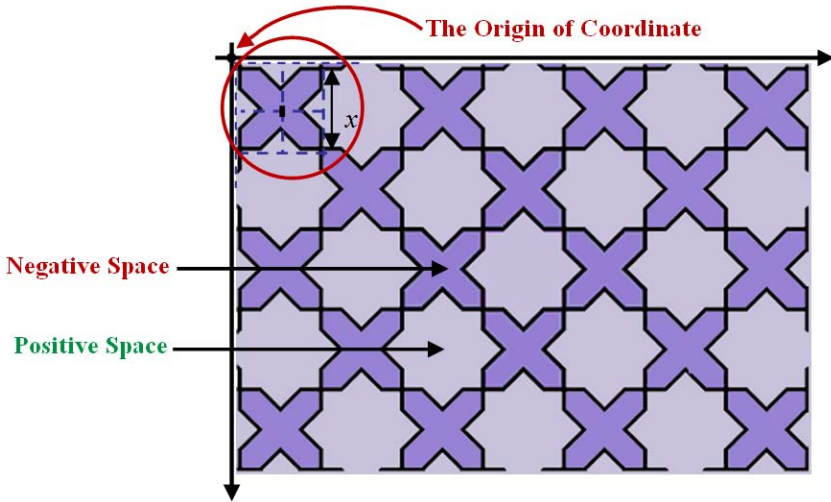


Fig. 2 Complete sketch made of by sample pattern repetition.

If two or more patterns join each other in some ways, they are called *Node*. The determined pattern in fig. 3 is called “*Bracelet*” which forms positive space of the sketch in fig. 2. The negation space of the sketch in fig. 2 is called “*Octa-Shamseh*”.

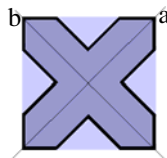


Fig. 3 The initial “*Bracelet*” pattern to create fig. 2 (*a* and *b* are the symmetry axes)

Noticing the pattern utilized in fig. 3, it can be understood that the pattern is made of a smaller parts which is shown in figure 4-a. and called fundamental region.

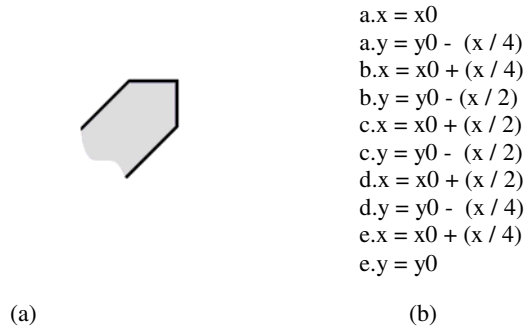


Fig. 4 Sketch Analysis via the fundamental region

This figure has five vertexes (a, b, c, d, e), the coordinate of which can be calculated easily in terms of the x as figure 3-b. The parameter x is the *side length* of the tetragon which is determined in fig. 2, the second and third parameters (x_0, y_0) are tetragon centre coordinates which depend on the origin of the coordinate and parameter x .

Fig. 5 shows the steps to find vertexes of fig. 4. This is done by procedure *FindVertex*. As a note it is not needed to determine all five vertex; in fact, if the vertex a and b are available, so vertexes d and e could be determined via “*reflective symmetry*” to the line oc in fig. 4.

```

Pocedure FindVertex (x , x0 , y0);
Begin
  VertexA0.X := x0
  VertexA0.Y := y0 - (x div 4);
  VertexB0.X := x0 + (x div 4);
  VertexB0.Y := y0 - (x div 2);
  VertexC0.X := x0 + (x div 2);
  VertexC0.Y := y0 - (x div 2);
  VertexD0.X := x0 + (x div 2);
  VertexD0.Y := y0 - (x div 4);
  VertexE0.X := x0 + (x div 4);
  VertexE0.Y := y0;
End;

```

Fig. 5 Vertex Calculating Procedure

There are three input parameters for procedure *FindVertex*. The first parameter x is the length of sub-tetragon in fig. 2 which is extracted in fig. 3. The second and third parameters are $x0$ and $y0$ respectively which are center of tetragon based on original coordinate axes.

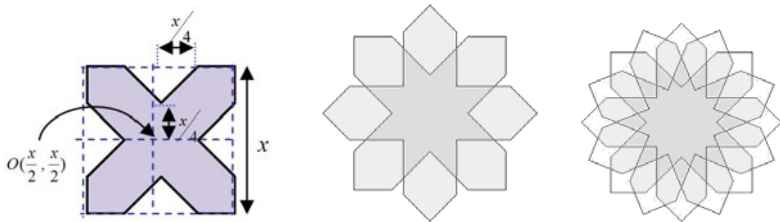
In general, in order to create an Islamic Geometric Pattern, there are three main steps that should be taken in to account.

1. Finding the basic sketch of which the whole pattern is made of.
2. Calculating relevant coordinate vertex for all basic sketches found in step 1.
3. Determining an appropriate arrangement system to create a complex geometric pattern, using basic sketches, found in step 1.

As a sample, we are going to introduce two most famous patterns called “*Star*” and “*Rosettes*”.

3.2 Rosettes

Assuming fig. 4-a as the fundamental region and performing the top three steps, fig. 6-a will be provided, with a graphical representation that shows how a more sophisticated patterns could be approached. Each individual region could be expanded by rotating and repetition of first sketch with different step angles of 90° , 45° and 22.5° . After obtaining coordinates of vertices for fig. 4 and drawing its sides, then rotating each vertex equal to 90° degree around the centre of quadrangle and repeating it, we can reach figure 6-a. This is called *rotational symmetry*. The procedure related to rotational symmetry is given in fig. 7.



- (a) Vertex rotation and repetition based on fig. 2-a in a surrounding tetragon and 90° step angle.
- (b) Vertex rotation and repetition with a 45° step angle
- (c) Vertex rotation and repetition with a 22.5° step angle

Fig. 6 Complicated patterns using rotational symmetry

```

Procedure Rotation_O (var p : TPoint ; const teta : Real );
var
  xx , yy : Variant ;
Begin
  xx := p.X × Cos(teta) - p.Y × Sin(teta) ;
  yy := p.Y × Cos(teta) + p.X × Sin(teta) ;
  p.X := xx ;
  p.Y := yy ;
End;
Procedure Rotation (var p : TPoint ; const teta : Real ; x , y : integer);
var
  temp : TPoint ;
Begin
  temp.X := p.X - x ;
  temp.Y := - p.Y + y ;
  Rotation_O (temp , teta) ;
  p.X := temp.x + x ;
  p.Y := - (temp.Y - y) ;
End;

```

Fig. 7 Rotational symmetry procedure around the centre point of (x,y)

Procedure *Rotation_O* in fig. 7, rotates p equal to $teta$ degrees around source coordinate axes. Here parameter p could be each of vertex points in fundamental region e.g. a, b, c, d, e in fig. 4. In this procedure, $teta$ and p are input parameters. Then second procedure-*Rotation*, rotates point p around (x,y) for $teta$ degrees and uses procedure *Rotation_O* to perform it. In this procedure $p, teta, x$ and y are input parameters.

If we transfer vertexes of fig. 3 around the centre of a $2x$ length tetragon, then rotate them with a 90° angle, so fig. 4-a will be obtained. Performing a similar manner with different repeating angles could be applied to reach the desired patterns. Looking back on fig. 2 it can be seen that the figure is provided by the transferring fig. 6-a with transfer vector of $(2x, 0)$. The negative and positive spaces in the existing sketch are determined in fig. 2. If fig. 6-a transferred with another transfer vector, other sketches also could be produced. This case depicts the robustness and advantage of suggested method, capable of generating wide diversity of forms just by a single fundamental form. In a similar fashion, we can reach to fig. 6-b if we repeat the vertexes eight times with a 45° rotating angle. Likewise, if we perform it 16 times with a 22.5° step angle, fig. 6-c will be produced.

In general, if we symmetrically rotate fig. 4 with the degree of $2\pi / n$, then the result will be a n -petals rosette which is surrounded in a regular n -gone.

Fig. 8 shows the relevant procedure to create a rosette flower. In this procedure, x is the first parameter for procedure *CreateRosette* and the *side length* of tetragon


```

Procedure CreateRosette (const x , x0 , y0 , n : Integer) ;
Var
  i : integer ;
  teta : Real ;
Begin
  teta := (2 * Pi) / n;
  for i := 1 to n do
    Begin
      MoveTo (VertexA0.X , VertexA0.Y) ;
      LineTo (VertexB0.X , VertexB0.Y) ;
      LineTo (VertexC0.X , VertexC0.Y) ;
      LineTo (VertexD0.X , VertexD0.Y) ;
      LineTo (VertexE0.X , VertexE0.Y) ;

      Rotation (VertexA0 , teta , x0 , y0) ;
      Rotation (VertexB0 , teta , x0 , y0) ;
      Rotation (VertexC0 , teta , x0 , y0) ;
      Rotation (VertexD0 , teta , x0 , y0) ;
      Rotation (VertexE0 , teta , x0 , y0) ;
    End ;
  End ;

```

Fig. 8 Rosette Creation Procedure

which is introduced in fig. 2. Second and third parameters- (x_0, y_0) are coordinate centre of the tetragon that was described in fig. 4. (x_0, y_0) is the centre of regular n -gon which surrounds the rosette. n the last parameter, determines the number of petals. Finally, given these four parameters a n -petals rosette would be created.

Although procedure *CreateVertex* works for n -angles with 2^i sides while $i > 1$, it doesn't work properly for a limited number of flowers used in Islamic patterns. A revised version of this procedure-*FindVertex* is developed as fig. 9 to cope with this limitation to produce n -petals flowers.

By adding parameters b_1, b_2, b_3, b_4 the function becomes more flexible to find vertex points of the flower and more powerful to create n -petals flowers. Fig. 10 shows some selected samples. All depicted patterns in this paper are real outputs from our developed software-GeoPat which is written in Delphi with all above procedures built in.

Procedure FindVertex_Ex (const x , x0 , y0 , b1 , b2 , b3 , b4 : integer);

Begin

VertexA0.X := x0 + b1 - b2;

VertexA0.Y := y0 + b2;

VertexB0.X := x0 + b4;

VertexB0.Y := y0 - b3;

VertexC0.X := x0 + (x div 2);

VertexC0.Y := y0 - (x div 2);

VertexD0.X := x0 + b3;

VertexD0.Y := y0 - b4;

VertexE0.X := x0 - b2;

VertexE0.Y := y0 - b1 + b2;

End;

Fig. 9 Extended form of Procedure FindVertex (Fig. 5.)

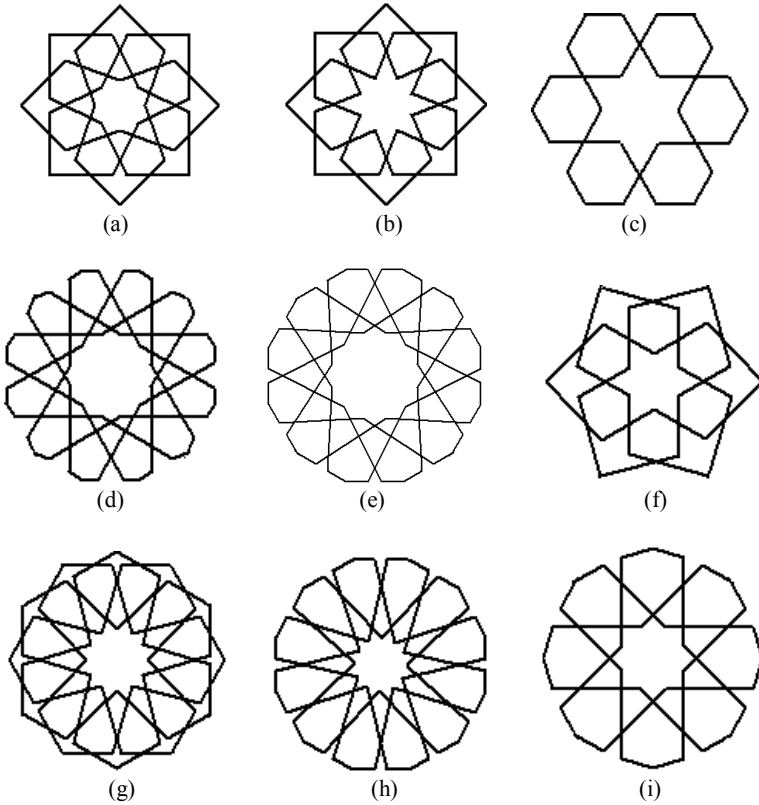
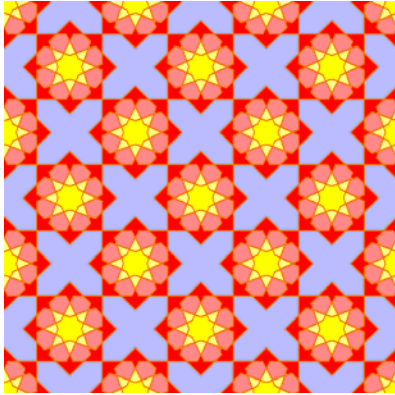


Fig. 10 Some basic fundamental patterns

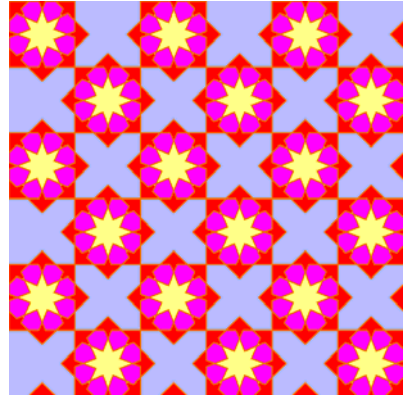
If we extend the patterns of fig. 10 each of which with different transformation vector, then various textures could be created. Some sample textures are provided as fig. 11 and 12.

The textures in fig. 11 and 12 are created based on a feature from fig. 10. In addition by changing the transmission vector, some other textures can be created.

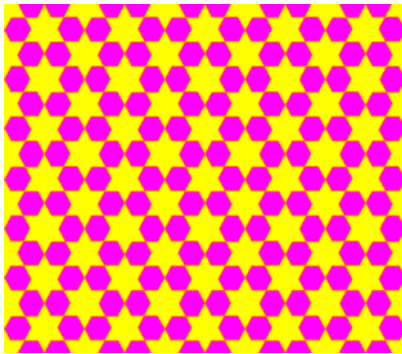
In GeoPat, The function Foodfill is considered to colorize the pattern, this option could be found in menu options.



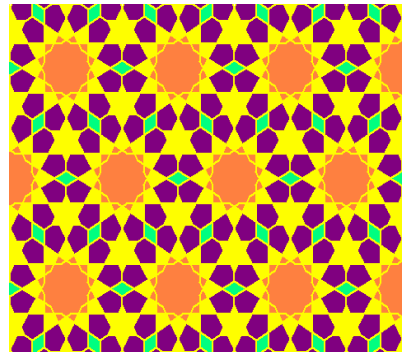
(a)



(b)



(c)



(d)

Fig. 11 Created patterns via transform of fig. 10-a, 10-b, 10-c and 10-d respectively

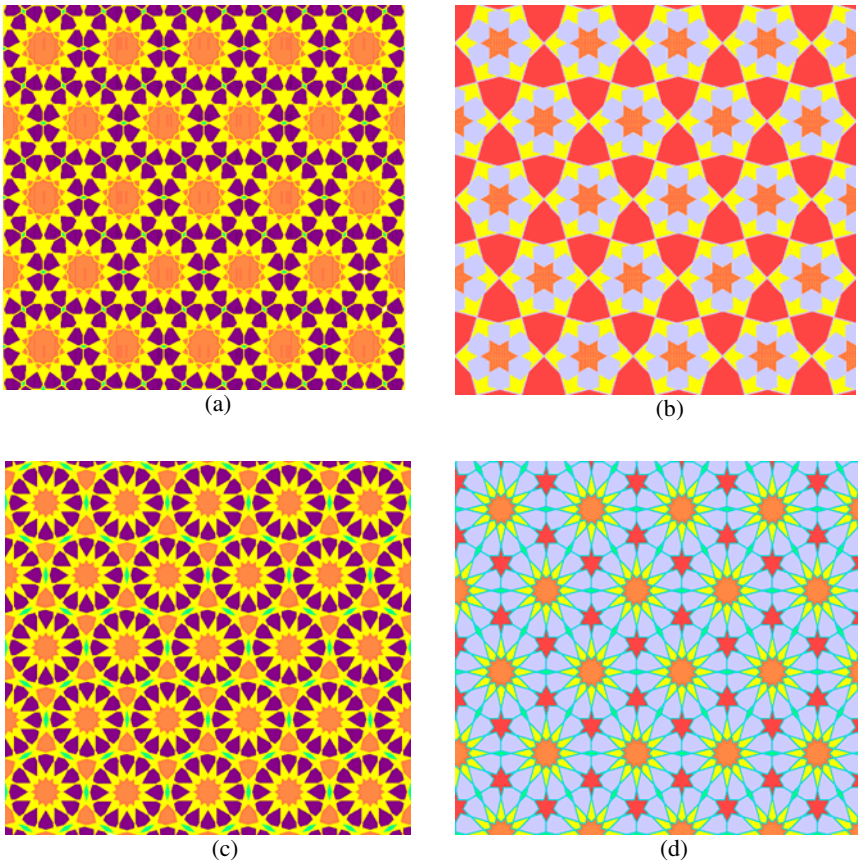


Fig. 12 (a) transformed texture via 10-e (b) Transformed texture via 10-f (c) Transformed texture via 10-g (d) transformed texture via 10-h

In order to have an initial sketch so to develop a texture we can perform tiling methods. Utilizing different tiling methods will creates different textures. Fig. 13 depicts how a tiling can create a texture. In fact, fig. 13-b is the same as fig. 12-d. actually tiling shows the transmission vectors in a clearer manner.

Fig. 13 is a result of expanding a tiling via a regular dodecagon. As can be seen there are also some regular triangles among these dodecagons. Filling the empty spaces of patterns by such triangles, we can reach more complex textures, similar to figure 14.

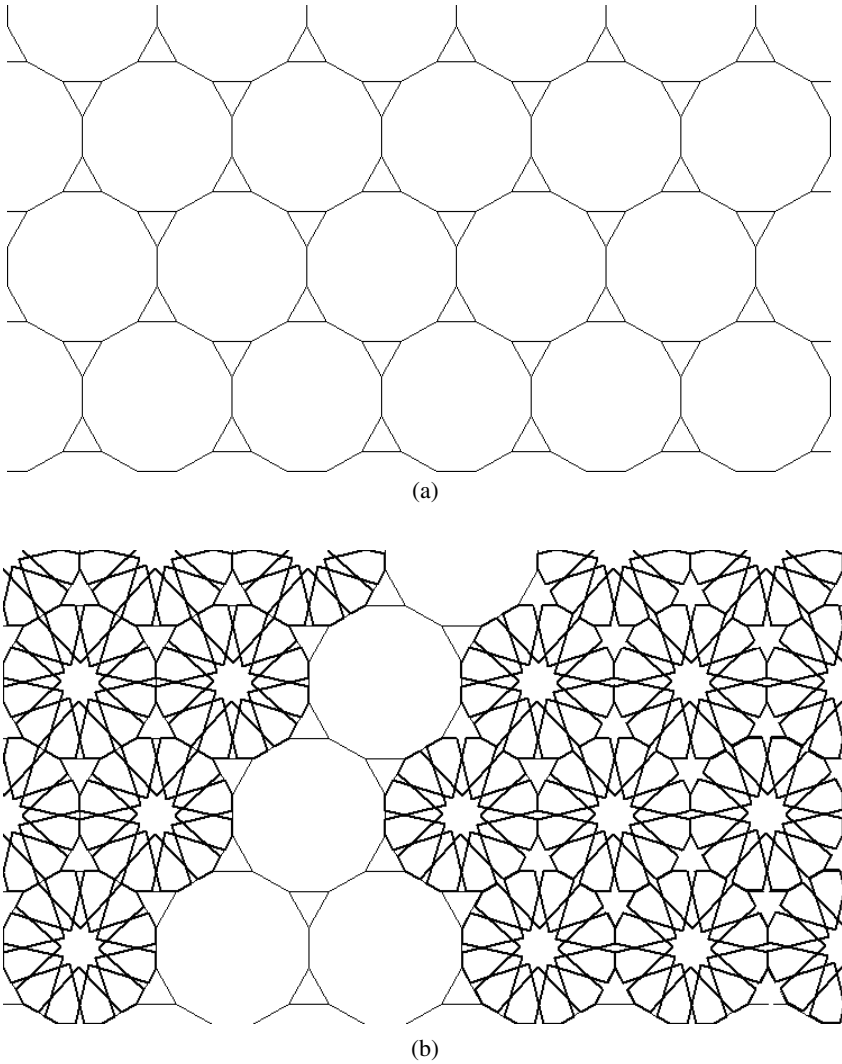
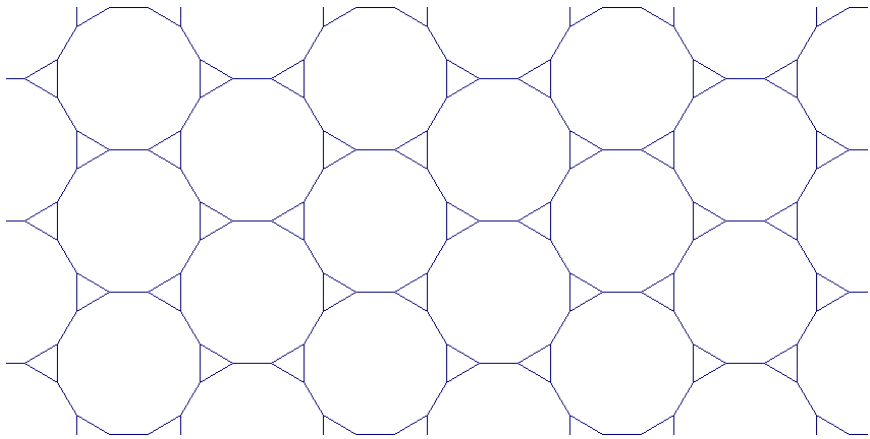
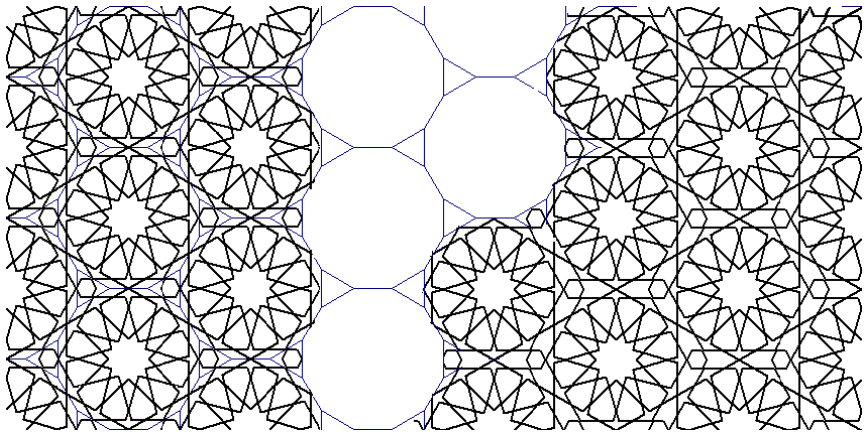


Fig. 13 (a) a regular dodecagon tiling sketch (b) a Developed texture by combining pattern 10-h and regular dodecagon tiling method.

In fig. 14, the regular dodecagon tiling is filled by patteen 10-g. Then regular triangles spaces are filled by a regular hexagon so in this way no irregular free space will remain.



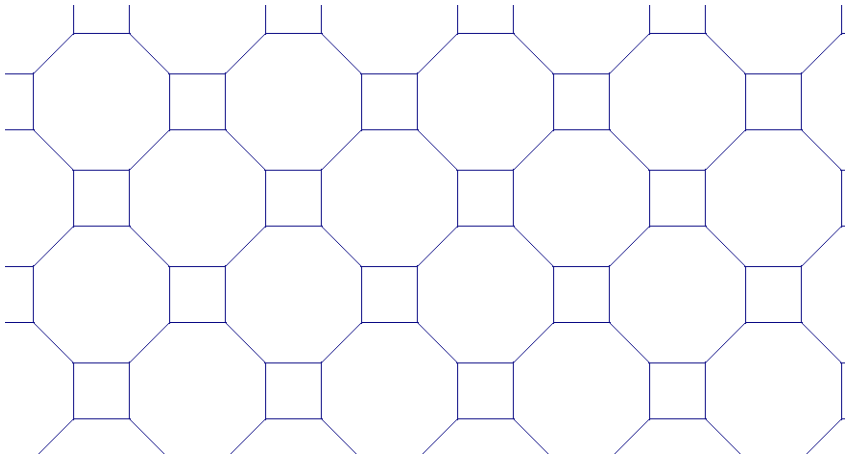
(a)



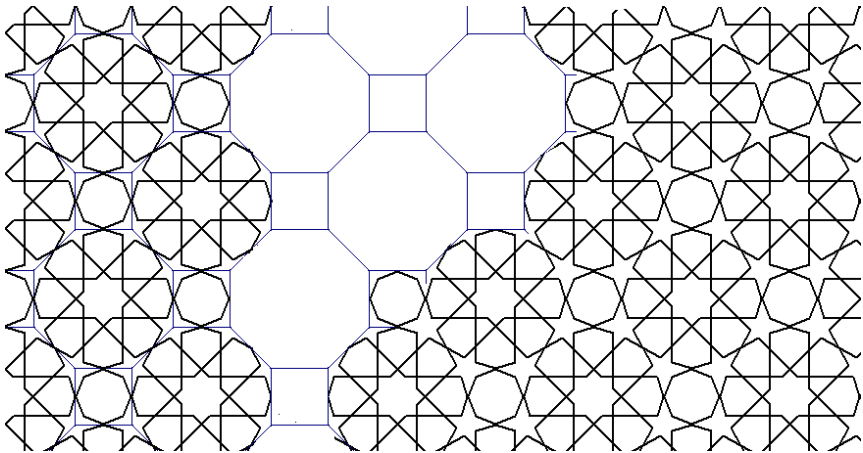
(b)

Fig. 14 (a) Regular dodecagon tiling layout (b) Final pattern combining 10-g and dodecagon tiling

Fig. 15-a shows a tiling pattern via a regular octagon. We use this pattern to produce the texture from fig. 17-d. In this layout, a regular tetragon is included within regular octagons. By expanding pattern 10-i the free space of tetragons will be filled by some regular octagons.



(a)



(b)

Fig. 15 (a) Regular octagon tiling pattern (b) Final texture mixing 10-i and regular octagon tiling pattern

In fig. 16-a a complex tiling that is made from regular dodecagon and tetragon can be seen. In this complex picture some triangles can be seen as well.

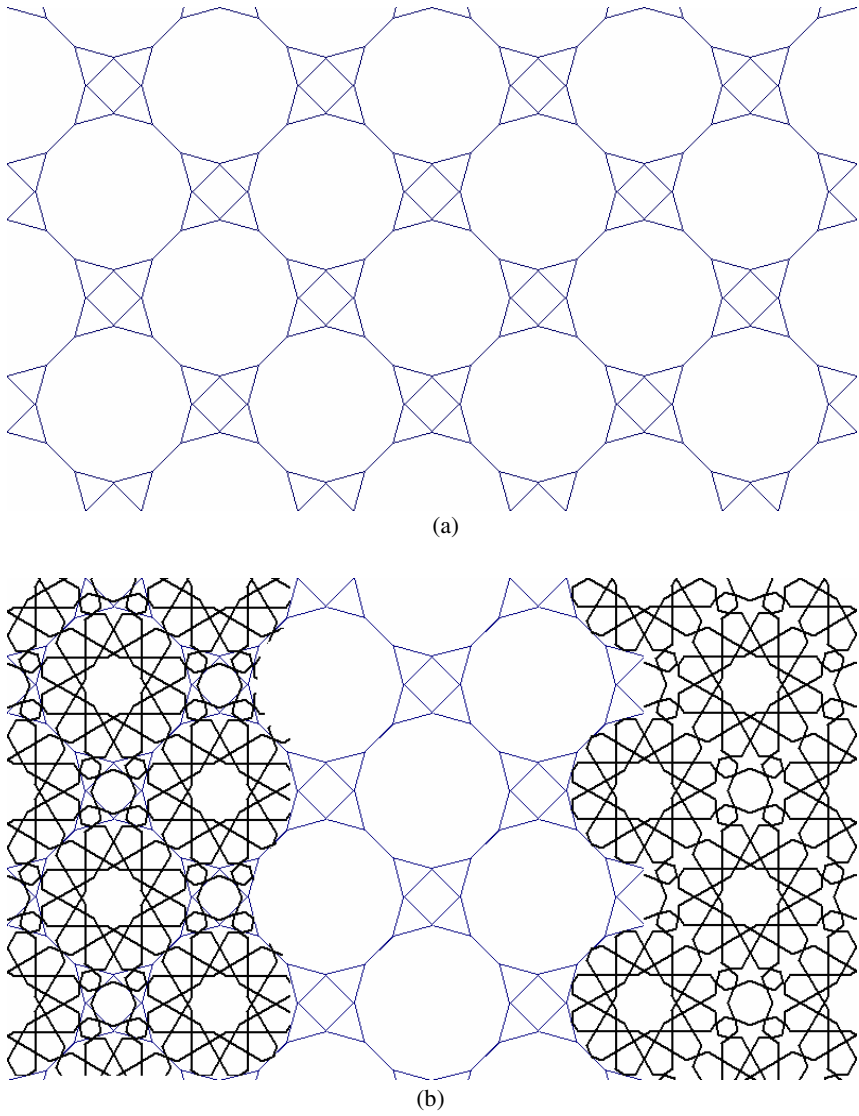


Fig. 16 (a) Hybrid tiling using regular dodecagons and tetragons (b) Final pattern via combining 10-d and regular dodecagons and tetragons

Fig. 16-a represents a hybrid tiling pattern, made of dodecagons and tetragons. Combining these two regular polygons plus some extra triangles has made a nice complex pattern.

In fig. 16-b by combining and filling “dodecagons with 10-d” and “tetragons with octagons” and then “triangle with hexagon” we can reach Fig. 17-a. These are some new textures made of with more than a pattern.

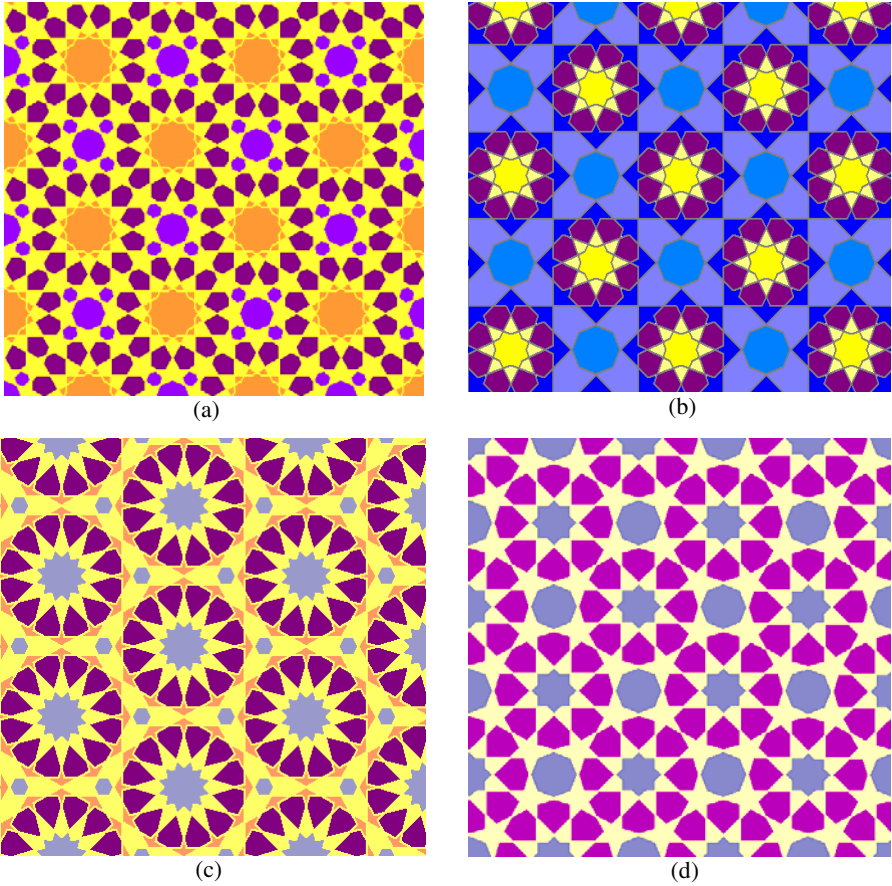


Fig. 17 (a) Hybrid texture via 10-d, regular hexagons and octagons (b) Texture made up of transformed 10-a and regular octagons (c) texture from transformed 10-g and regular hexagons (d) texture form transformed 10-I and regular octagons

3.3 Stars

Other patterns that are abundantly observed in Islamic Geometric patterns are the existing patterns in fig. 18 which are classified in the Islamic star set. The existing pattern in fig. 18-a is made of the repetition of a smaller part shown in fig. 19.

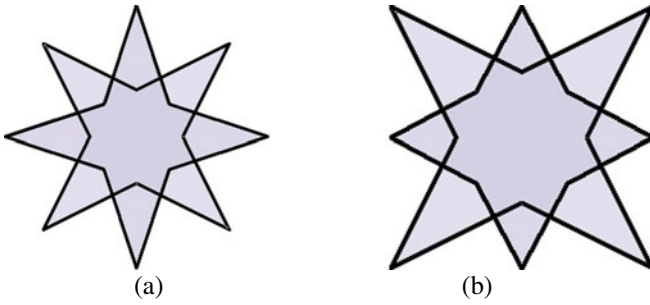


Fig. 18 Octagonal Stars

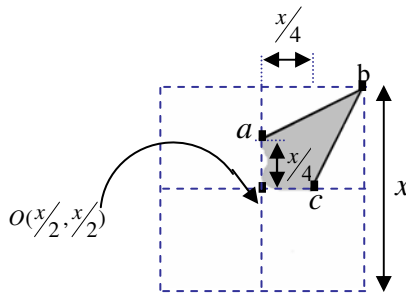


Fig. 19 Initial part of fig. 5

As we assumed (x_0, y_0) as the centre of coordinate system in fig. 3, now we produce fig. 19 based on parameter 'x'. Below is the relevant coordinate axes for points a, b and c :

- $a: (x_0, (y_0 - (x/4)))$
- $b: ((x_0 + (x/2)), (y_0 - (x/2)))$
- $c: ((x_0 + (x/4)), y_0)$

Joining a to b and b to c , fig. 19 will be produced. Fig. 18-a is the rotated form of fig. 19 in a 45° angle for all points of a, b and c .

In fig. 18-b points a and c have the 45° rotation angle, but point b will be moved on the tetragon sides. It means that points a and c have periodic symmetry and point b have transfer symmetry on the tetragon sides.

In fig. 19, if points a and c join to each other and rotate with a 90° angle around the tetragon center of (x_0, y_0) , then fig. 20 will be obtained. The same way is correct for fig. 4. That is, if the vertexes a and e join each other, then the provided figure will produce divergent sketches.

Fig. 21 shows a transfer vector used in different directions and length. Here, we prepared some diversity of sketches which are the extended version of fig. 18 and 20.

In fig. 21, horizontal transfer vector is $(x, 0)$ and vertical transfer vector is $(0, x)$. Considering all provided sketches so far, we just discussed about sketches which obtained as a result of symmetry and fundamental pattern repetition on the surface.

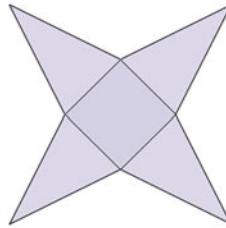
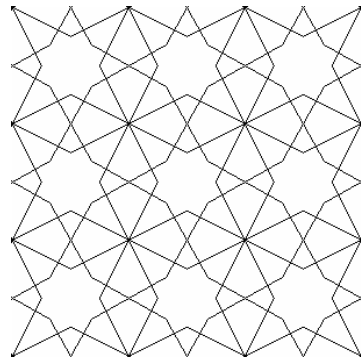
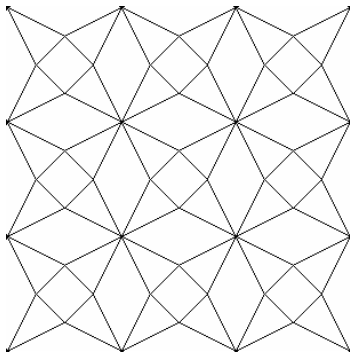


Fig. 20 Reorganized form of Fig. 19 by jointing points a and c , plus a 90° rotation.



(a) Extended version of fig. 7.
 $(x, 0)$: horizontal transferred vector
 $(0, x)$: vertical transferred vector

(b) Extended version of fig. 5.
 $(x, 0)$: horizontal transferred vector
 $(0, x)$: vertical transferred vector

Fig. 21 Transfer vector produced in different directions and length

Fig. 21 indicates some more complex sketches which are provided as the result of combination and expansion of multi-patterns on the surface. Fig. 22-a is an extended and combined version of fig. 6-a and 18-a, with the transfer vector of $(2x, 0)$ for horizontal expansion, and transfer vector of $(0, 2x)$ for vertical expansion.

Fig. 22-b is the result of combination and expansion for fig. 6-a and 20. Similarly, fig. 22-c is the outputted result of combination and expansion which is applied on fig. 6-a and 18-b.

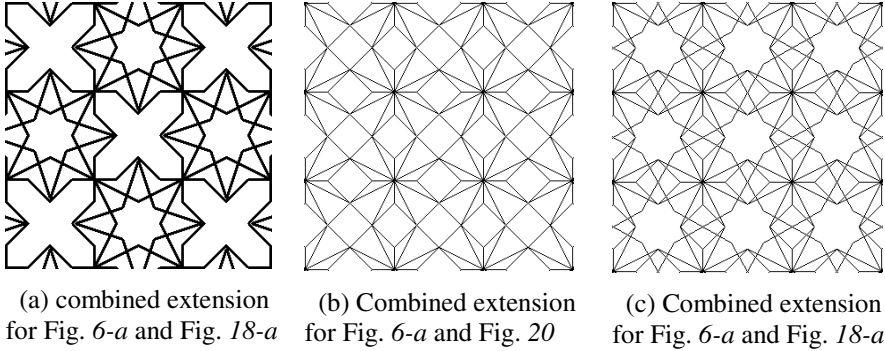


Fig. 22 Complex sketches as the result of “*expansion*” and “*combination*” applied on different patterns

3.4 Other Fundamental Pattern

Another fundamental region to create diversity patterns is shown in fig. 23. This figure is composed of an octagon and four diagonal lines. By changing the vertices of this figure some other fundamental regions so a wide variety of new patterns could be created. Here, one of the lines is highlighted by the vertices a, b and c . Similar to the method mentioned for fig. 4, and using parameter x , the position and coordinate of a, b and c is reachable.

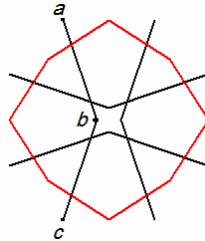


Fig. 23 A new sample fundamental region

After producing the fundamental forms, some samples of which we have mentioned, the sketches should be made with fundamental forms. As mentioned before, transfer vectors are used to produce sketches.

In fig. 24, some other patterns are given that are originated from fundamental region of fig. 23. The differences of these six patterns are rotation angle of vertices and transform vector.

In fig. 25, another pattern is provided. This sketch is made up of the extended fundamental pattern in fig. 26. Analyzing fig. 26-a, we will find it as the forming part of fig. 26-b. In order to reach fig. 26-b, it just needed to transform the coordinate vector of points b, c, d in fig. 4 toward the given direction in fig. 27.

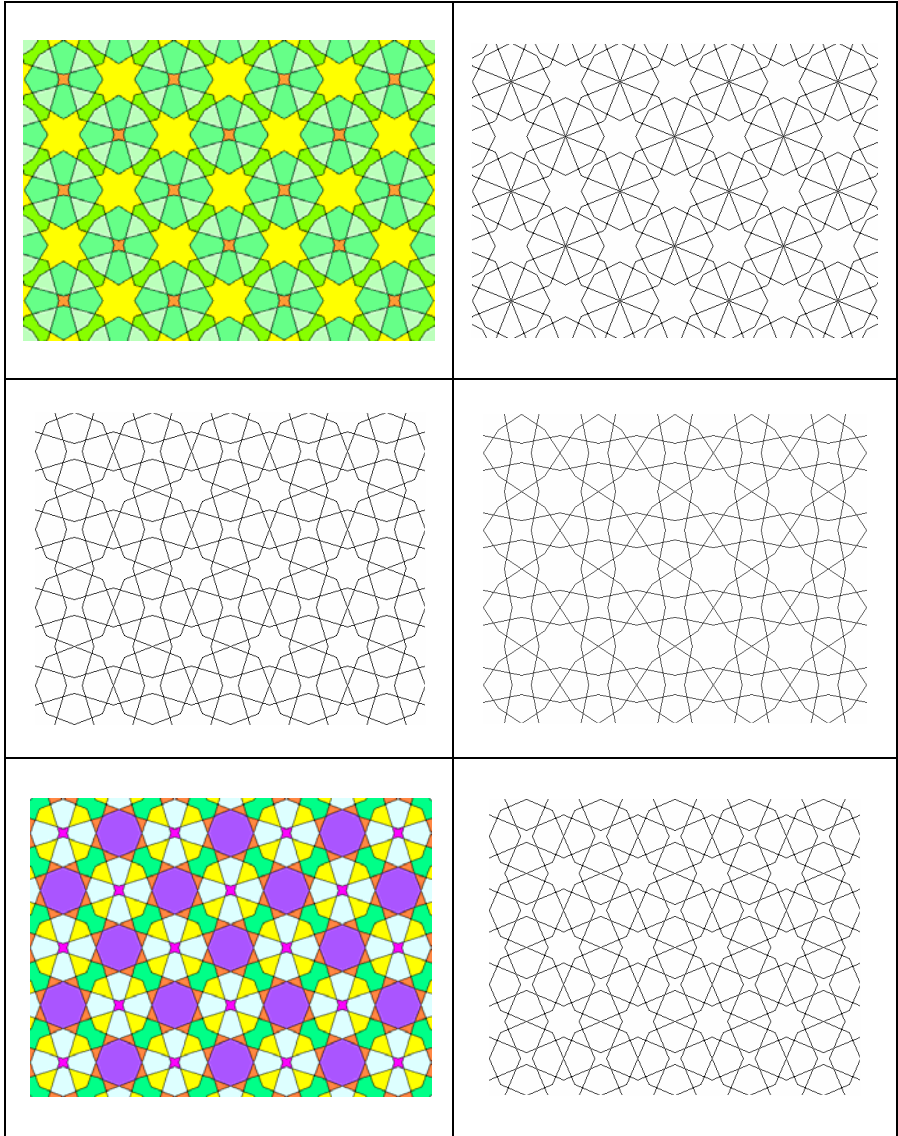


Fig. 24 Some new pattern beginning with fig. 23

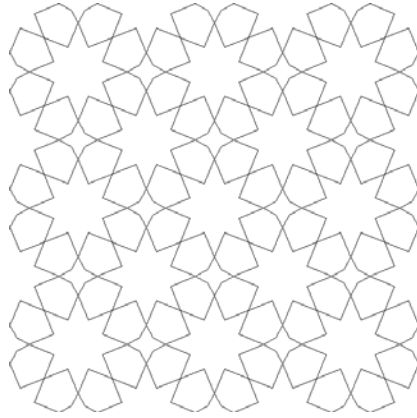


Fig. 25 Another Complex Sketch made up of fig. 26.

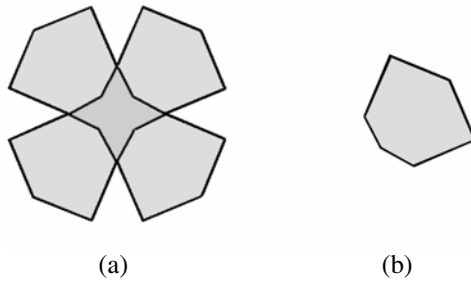


Fig. 26 Another extension of fig. 4

In fig. 27 it can be seen how fig. 4, 19 and 26-b can be accorded and how these forms can be transformed to each other by replacement of points *b*, *c* and *d*. So, in that way we can easily create both rosettes and stars just by one procedure.

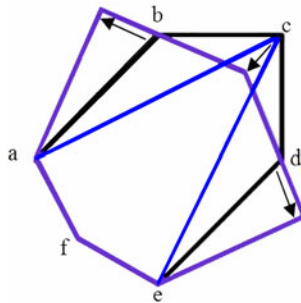


Fig. 27 Accorded form for fig. 4, 19 and 26-b.

Utilizing the given method in fig. 27, a more flexibility will be gained to create more complex patterns. As an illustration, via this method fig. 17-d can be converted to fig. 28.

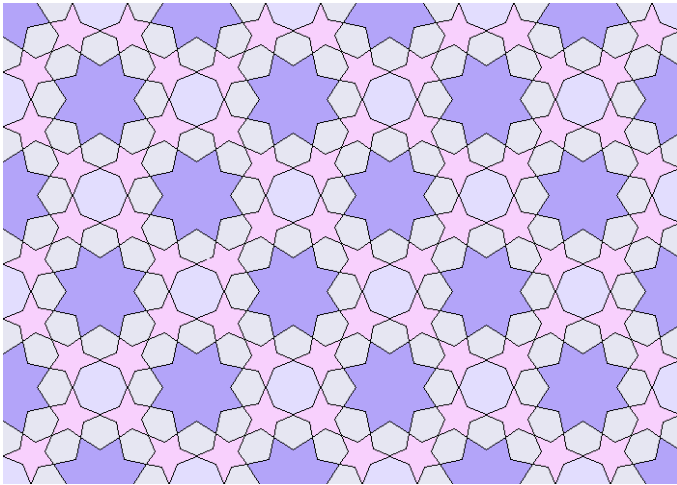


Fig. 28 Converted form of fig. 17-d

3.5 Figure Transformation

Figure transformation is one of the ornamental sketch styles created by William Huff and then generalized by Dongles Hofstadter [17]. In fact, transformation means starting from a figure, then changing the figure and finally transforming it to another figure.

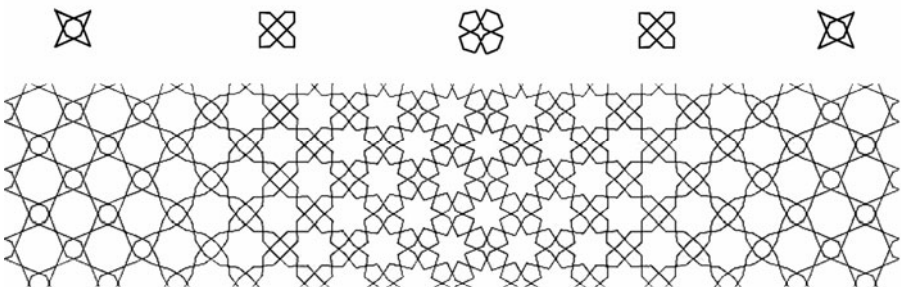


Fig. 29 A Figure Transform Sample for texture development

Fig. 29 illustrates a sample transformation. Applying the proposed method, we can easily change the vertexes of the coordinate in the fundamental figure, so we are capable of creating such a transformation and to make the fundamental area.

Fig. 30 is the GUI of the written software in Delphi.

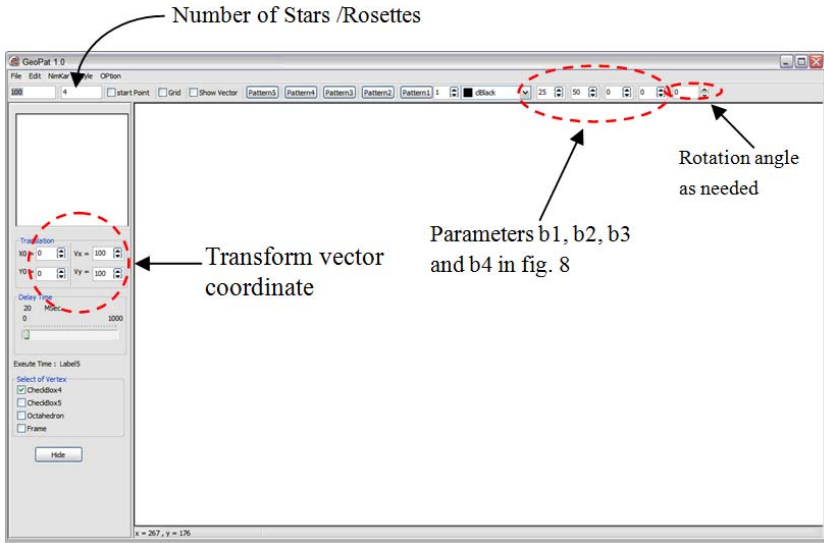


Fig. 30 Graphical User Interface to implement ornamental sketch

By selecting the desired sketch from *Style* menu and then entering the required parameters, the overall form of sketch will be created. Then by defining and entering the transformation coordinate vector, the target pattern will be created.

Fig. 31 shows a sample pattern and its relevant parameters

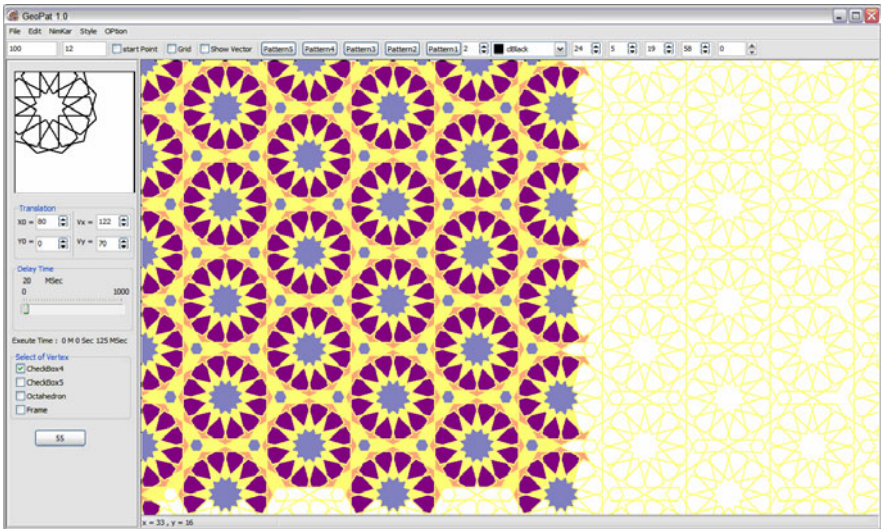


Fig. 31 A sample pattern created by GeoPat version 1.0

4 Conclusions

In this article we offered a new algorithm by which we created several patterns of Islamic Geometric patterns. Introducing a novel point of view for complicated patterns is the prominence of the proposed method. In addition, utilizing basic geometric concepts, the method pursue for individual fundamental sketches, so the computerized implementation would be systematic and easier to understand.

Most of the other's presented methods either focus on generating new patterns or try to map traditional patterns. In contrast, our method not only is able to create and develop sophisticated patterns, but also can expand both traditional and contemporary shapes and patterns.

The proposed method is capable of producing diversity geometric patterns. Having a new view to complicated patterns and sketches is the strength and the highlighted point of this method. The method uses simple individual figures and approaches to a highly sophisticated pattern through some basic geometric concepts.

Most of currently available methods are trying either to create new patterns or just to find a retrieval method to create traditional graphs; however, our method is capable to working on both, that is, possibility of creating new patterns as well as traditional patterns.

As the future work, we will work on 3D colourful Islamic Geometric patterns and try to provide a hybrid algorithm based on the top mentioned method, other researcher's endeavours and image processing techniques. Our objective is to cope with disadvantages and defect of ordinary algorithms in the field. One other thing we are going to improve is filling the existing gaps in the sketches. The authors are currently working on the software to aim above.

References

- [1] Kaplan, C.S.: Computer generated Islamic Star Patterns. In: Sarhangi, R. (ed.) Bridges 2000 Proceedings (2000)
- [2] Ahmad, M., Aljamali, Banissi, E.: Grid Method Classification of Islamic Geometric Patterns. In: Proceeding of WSCG 2003, February 3-7. UNION Agency – Science Pres, Plzen (2003)
- [3] Bonner, J.: Geodazzlers-Eighteen Ornamental Polyhedra for Easy Construction. Design Science Toys Publishing, ASIN: B0010OMLLQ (1997)
- [4] Kaplan, C.S., Salesin, D.H.: Islamic Star Patterns in Absolute Geometry. ACM Transactions on Graphics (TOG) archive 23(2), 97–119 (2004)
- [5] Lee, A.J.: Islamic Star Patterns. In: Grabar, O. (ed.) Muqarnas IV: An Annual on Islamic Art and Architecture, E.J. Brill, Leiden (1987)
- [6] Ostromoukhov, V.: Mathematical Tools for Computer-generated Ornamental Patterns. In: Hersch, R.D., André, J., Brown, H. (eds.) RIDT 1998 and EPub 1998. LNCS, vol. 1375, pp. 193–223. Springer, Heidelberg (1998)
- [7] Bonner, J.: Three traditions of self-similarity in fourteenth and fifteenth century Islamic geometric ornament. In: Proceedings ISAMA/Bridges, Granada, pp. 1–12 (2003)

- [8] Bonner, J.: *Islamic Geometric Patterns: Their Historical Development and Traditional Methods of Derivation*. Unpublished (2000)
- [9] Hankin, E.H.: *The Drawing of Geometric Patterns in Saracenic Art*. *Memoirs of the Archaeological Society of India* 15, LCCN: 96909213 (1925)
- [10] Salman, A.S., Abas, S.J.: *Symmetries of Islamic Geometrical Patterns*. World Scientific Pub. Co., Singapore (1995) ISBN: 9810217048
- [11] Dewdney, A.K.: *The Tinkertoy Computer and Other Machinations*, pp. 222–230. W. H. Freeman Publishing, New York (1993) ISBN: 071672491X
- [12] Abas, S.J., Salman, A.S.: Geometric and group-theoretic methods for computer graphics studies of Islamic symmetric patterns. *Computer Graphics Forum* 11(1), 43–53 (1992)
- [13] Grünbaum, B., Shephard, G.C.: *Interlace patterns in Islamic and Moorish art*. In: *The visual mind: art and mathematics*, pp. 147–155. MIT Press, Cambridge (1993)
- [14] Dispot, F.: *Arabesque Studio*, a Java Tool (2002),
<http://www.wozzeck.net/arabesque/index.html>
- [15] Valiente, J.M., Albert, F., Gomis, J.M.: *A Computational Model for Pattern and Tile Designs Classification Using Plane Symmetry Groups*. In: Sanfeliu, A., Cortés, M.L. (eds.) *CIARP 2005*. LNCS, vol. 3773, pp. 849–860. Springer, Heidelberg (2005)
- [16] Lalvani, H.: *Coding and generating complex periodic patterns*. *Journal: The Visual computer* 5(4), 180–202 (1989) ISSN: 0178-2789
- [17] Hofstadter, D.: *Metamagical Themas: Questing for the Essence of Mind and Pattern*. Basic Books (1986) ISBN: 0-465-04566-9
- [18] Lu, P.J., Steinhardt, P.J.: *Decagonal and Quasi-Crystalline Tilings in Medieval Islamic Architecture*. *Journal of Science* 315(5815), 1106–1110 (2007)
- [19] Rasouli, P., Bastanfard, A., Rezvanian, A., Jalilian, O.: *Fast Algorithms for Computer Generated Islamic Patterns of 8-ZOHREH and 8-SILI*. In: Huang, Y.-M.R., Xu, C., Cheng, K.-S., Yang, J.-F.K., Swamy, M.N.S., Li, S., Ding, J.-W. (eds.) *PCM 2008*. LNCS, vol. 5353, pp. 825–829. Springer, Heidelberg (2008)
- [20] Cromwell, P.R.: *The Search for Quasi-Periodicity in Islamic 5-fold Ornament*. *The Mathematical Intelligencer* 31(1) (January 2009) ISSN: 0343-6993
- [21] Grünbaum, B., Shephard, G.C.: *Tilings and Patterns*. W.H. Freeman, New York (1987)